

Hong Kong Mathematics Olympiad (2023/24)

Heats – Individual Event

香港数学竞赛 (2023/24)

初赛个人项目

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

The diagrams are not necessarily drawn to scale.

除特别指明外，所有答案须以数字的真确值表达，并化至最简。

不接受近似值。

所有附图不一定依比例绘成。

Part A

甲部

1. Find the sum of the factors of 2024.

求 2024 的所有因子之和。

2. If $a^{3y} = 729$, find the value of a^{-2y} .

若 $a^{3y} = 729$ ，求 a^{-2y} 的值。

3. A 6-digit number is formed by joining two identical 3-digit numbers, such as 256256 and 678678. Find the greatest common factor of these 6-digit numbers.

一个 6 位数由两个相同的 3 位数组组合而成，如 256256 及 678678。求这些 6 位数的最大公因子。

4. If $4^{x+3} - 47 = 193 + 4^{x+1}$, find the value of $(4^{x+3})(4^{x+1})$.

若 $4^{x+3} - 47 = 193 + 4^{x+1}$ ，求 $(4^{x+3})(4^{x+1})$ 的值。

5. In a right-angled triangle, the lengths of the medians from the vertices of the acute angles are 7 and 9. Find the length of the hypotenuse of the triangle.

在一直角三角形中，从锐角顶点所作的中线长度为 7 及 9。求三角形斜边的长。

6. Eric was born in 20th century (1901 – 2000), and he was y years old in the year of y^2 . Find his year of birth.

志伟生于 20 世纪 (1901 – 2000)，于 y^2 年时的岁数为 y 。求志伟的出生年份。

7. As shown in Figure 1, a common tangent touches a large and a small circles at P and Q respectively. Given that the two circles touch each other at C and their radii are 49 and 25 respectively, find the length of PQ .

如图一所示，一条公切线与一大圆及一小圆分别相交于点 P 及 Q 。已知该两圆相交于点 C 且它们的半径分别为 49 及 25，求 PQ 的长。

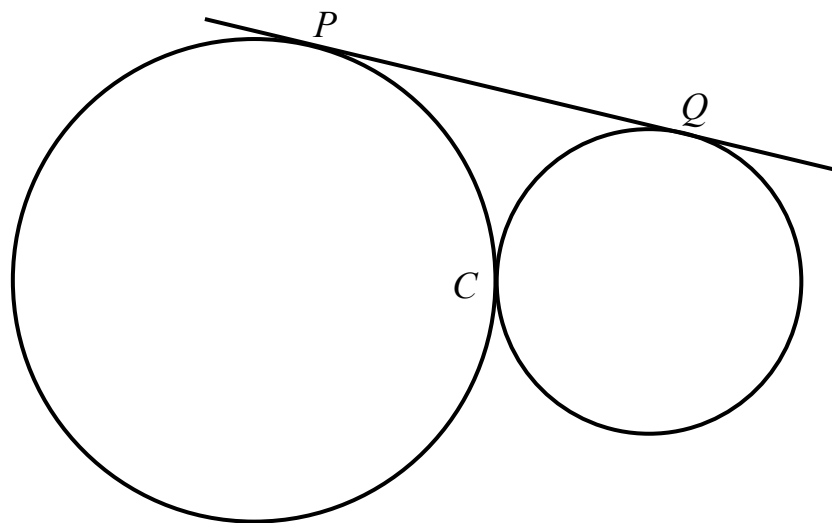


Figure 1

图一

8. As shown in Figure 2, $ABCD$ is a quadrilateral. If $\angle ABD = \angle CBD = 60^\circ$, $\angle ADB = 74^\circ$ and $\angle CDB = 53^\circ$, find the value of $\angle BAC$.

如图二所示， $ABCD$ 是一个四边形。若 $\angle ABD = \angle CBD = 60^\circ$ ， $\angle ADB = 74^\circ$ 及 $\angle CDB = 53^\circ$ ，求 $\angle BAC$ 的值。

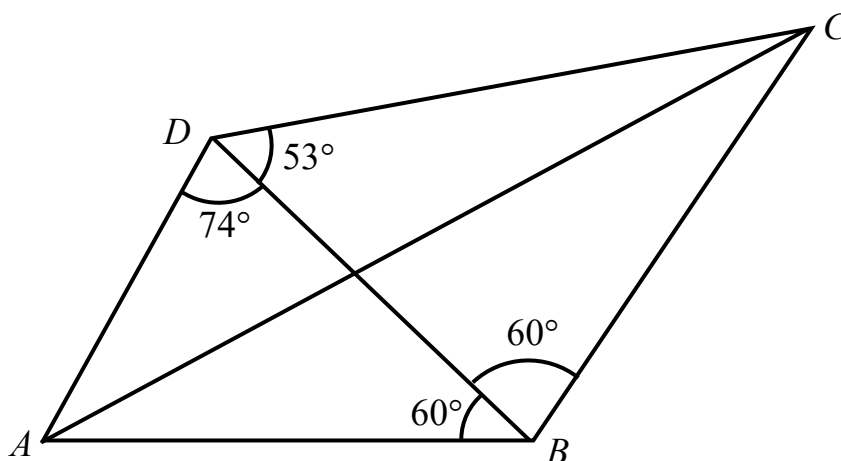


Figure 2

图二

9. Let a be a positive real number. If the system of equations $\begin{cases} (a+3)x + (a+2)y = 1 \\ (a-1)x - ay = 1 \end{cases}$ has no solution, find the value of a .

设 a 为正实数。若方程组 $\begin{cases} (a+3)x + (a+2)y = 1 \\ (a-1)x - ay = 1 \end{cases}$ 无解，求 a 的值。

10. Figure 3 shows the circle $ABCDEFGH$. Find the value of $a+b+c+d$.

图三所示为圆 $ABCDEFGH$ 。求 $a+b+c+d$ 的值。

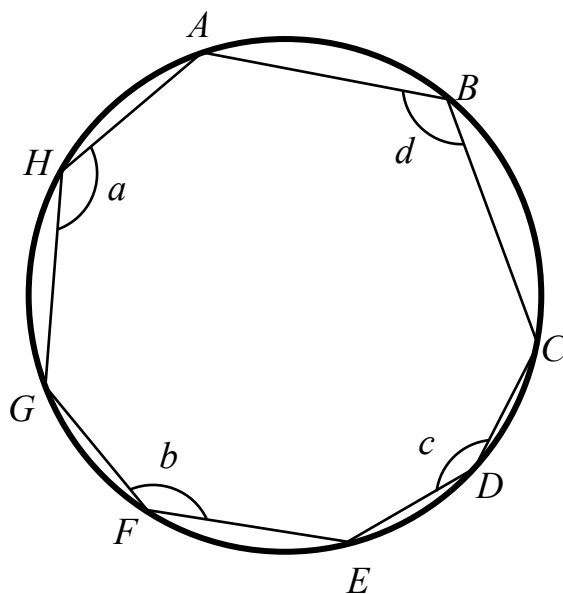


Figure 3

图三

Part B

乙部

11. If $1+2+3+\cdots+k$ is a perfect square N , where $N < 250\,000$, find the largest possible value of N .

若 $1+2+3+\cdots+k$ 的和为一完全平方 N ，其中 $N < 250\,000$ ，求 N 的最大可能值。

12. If the lengths of the three sides of a $\triangle ABC$ are 9, 10 and 17, find the radius of the circum-circle of $\triangle ABC$.

若 $\triangle ABC$ 的边长为 9、10 及 17，求 $\triangle ABC$ 外接圆的半径。

13. Find the value of $S = \frac{1}{2024} - \frac{3}{2024^2} + \frac{5}{2024^3} - \frac{7}{2024^4} + \frac{9}{2024^5} - \dots$.

求 $S = \frac{1}{2024} - \frac{3}{2024^2} + \frac{5}{2024^3} - \frac{7}{2024^4} + \frac{9}{2024^5} - \dots$ 的值。

14. In Figure 4, XY is a diameter of the circle with centre at O and radius 5 cm. XY intersects the chord AB at Q such that $\angle AQO = 90^\circ$ and $XQ = QO$. A semi-circle with diameter AB intersects XY at M . BM produced intersects the circle at C . Find the length of AC .

图四中， XY 是一个以 O 为圆心及半径为 5 cm 的圆的直径。 XY 与弦 AB 相交于点 Q ，使得 $\angle AQO = 90^\circ$ 及 $XQ = QO$ 。以 AB 为直径的半圆与 XY 相交于 M ，延线 BM 与圆相交于点 C ，求 AC 的长。

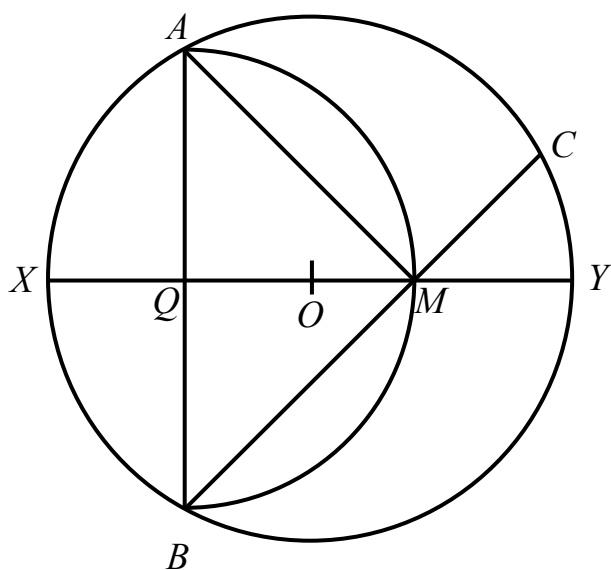


Figure 4

图四

15. In Figure 5, P and R are points on the circle C . AP is the tangent to C at P and AR intersects C at Q . If $QR = 10$ and $PA = 5\sqrt{3}$, find the length of AQ .
- 图五中，点 P 及 R 均在圆 C 上。过点 A 的切线与 C 相交于 P 及 AR 与 C 相交于 Q 。若 $QR = 10$ 及 $PA = 5\sqrt{3}$ ，求 AQ 的长。

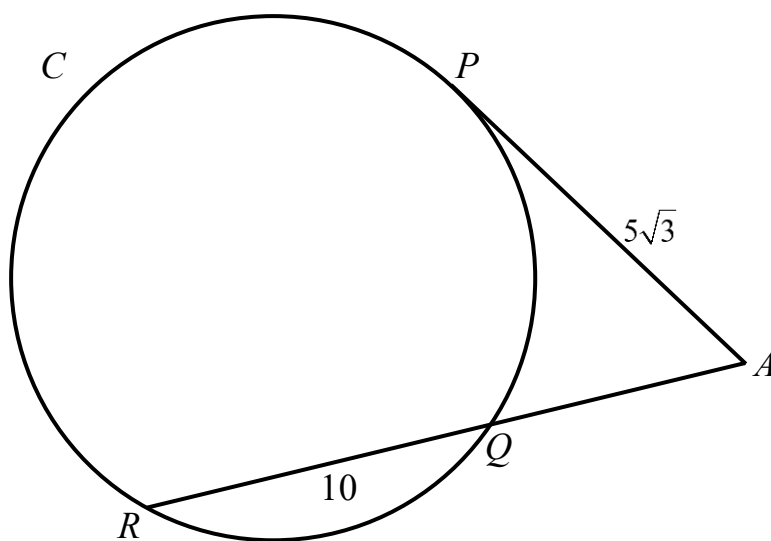


Figure 5

图五

END

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